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RESEARCH MEMORANDUM

HIERARCHICAL TRADE AND DOWNSTREAM INFORMATION

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Abstract

The purpose of this paper is to present a model of a trade economy in which the dominance of certain agents over others plays a major rôle. The agents are assumed to be organized in a hierarchical structure, in which the agents higher in the hierarchy dominate the agents lower in the hierarchy in the sense of price setting.

In this model the information structure, which determines the anticipated reactions of followers on a change of actions by their leader, is of crucial importance. We assume that agents correctly anticipate the actions of all the agents which are downstream from them in the hierarchical structure. Unfortunately existence of equilibria can only be proved under rather restrictive conditions.

Three examples are given in this paper. The first illustrates non-existence of equilibrium. The other two illustrate the impact of different hierarchical structures on the equilibrium outcomes of the economy.

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1 Introduction

It is generally acknowledged that one of the major flows in general equilibrium theory is the absence of a model of price formation. Recently there has been renewed interest in the modelling of competition in markets such that the structure of the market is taken into account. One of the most natural features of such a structure is that of dominance of one agent over another. In oligopoly theory producers may act as leaders with respect to the price taking consumers. In such case the dominance of the producer can be viewed as a natural consequence of his individual attributes, in particular the ability to produce and to supply a large share of the market, in comparison with the individual attributes of the other agents in the economy.

In other models of competition, however, dominance of one agent over another is not a natural consequence of his individual attributes, but rather a consequences of the trading rules, i.e., of the "rules of the game". In most of these cases the rules are describing some kind of institutional structure within the market, in which dominance of one agent over another is a social economic phenomenon. This is illustrated by referring to the traditional Stackelberg duopoly model, in which one producer dominates the other by some externally given rule. Aumann (1973) and Kalai et al. (1978) showed that in certain situations dominance over other agents is not profitable as is the case in the Stackelberg model.

In this paper we follow this line of research. We assume that the dominance, in the sence of price setting, of one agent over others is a result of a hierarchical structure on the market, that is, the dominance relations between the agents in the market is are designed top–down and result in several hierarchical levels. This type of hierarchical structures is presented in theories of industrial organization as given by Machup and Taber (1960) and Krelle (1976) and more recently in Robson (1990) and Weber (1989). In general equilibrium theory this line of research resulted in models of monopolistic competition with a flat hierarchical structure, e.g. in Negishi (1961), Arrow and Hahn (1971) and, more recently, Roberts (1987). In Gilles (1989) a model with a deeper hierarchical structure is given. In this paper we follow, in this respect, the line as developed in Gilles (1989).

We consider a finite pure exchange economy that consists of an ordered set of agents, of an individual attribute space, and of institutional rules specifying the relations between agents. The agents are ordered according to some hierarchical dominance pattern. This hierarchy is described by a directed graph in which an arc from one agent to another implies that the first agent dominates the second agent. The dominating agent is called the leader, whereas the dominated agent is called follower.

To circumvent complex situations arising from cycles and group descisions, we assume that the hierarchy graph in the economy is a tree. Hence, the number of trade relations is minimal with respect to the requirement that any agent is potentially connected to any other agent in the economy through a finite number of intermediary agents. This implies that if one destroys one of the trade relations in such an economy, one arrives at the situation in which there are two separate economies, which are not able to trade with each other through the given trade relations. One thus arrives at situations as described by Ioannides (1990) and Haller (1990) who study the patterns of trading groups that emerge in models with random trade relations between agents. In Spanjers (1991) specific situations in which the trade relations do not form a tree are analyzed.

We assume that each relation has the institutional characteristics of a monolopy, the leader in the relation sets the prices at which trade takes place, the follower chooses the quantities to be traded on the relation. The leader in a relation has the obligation to meet the quantities determined by the follower. So the agents of the lowest hierarchical level in the economy only act as price takers, while the agent of the highest hierarchical level only acts as a price setter. The agents of the "inbetween" hierarchical levels act as price takers with on the relation with their leader and act as price setters with respect to their followers.

An important question to be answered concerns the information that leading agents have about the reactions of following agents on the prices they set. There are several plausible rules to derive the conjectures of an agent from his knowledge of the individual characteristics of the agents and the institutional characteristics of the relations. This problem occurs since the hierarchical structure of the economy not only describes limitations on the possibilities to trade, but is also assumed to limit the possibilities of agents to communicate. As a consequence, agents may have limited access to information on the reaction patterns of other agents.

In one extreme case, the case of local information, we assume each agent knows nothing but the institutional characteristics of the relations he is a part of and of the relations between his direct followers and and their leaders, and the individual characteristics, being the initial endowments and the utility function, of himself and of his direct followers. Furthermore he is assumed to know the sum of trades of each of his followers with their followers, the prices set by for him by his leader(s), and the prices set for his (direct) followers by their leaders. This information structure, which is called the *local information structure,* is analysed in Spanjers e.a. (1991) for an economy with a hierarchy tree and in Spanjers (1991) for a different class of hierarchical structures.

The other extreme case is the *subgraph information structure* which describes the situation in which each agent knows the individual characteristics of each of the agents of which he is the (indirect) leader, knows the institutional characteristics of the relations between those followers and their leaders, knows the institutional characteristics of his own relations with his leaders, and, given the state of the economy, knows the actions of his leaders and those of the leaders of his (indirect) followers. This allows each agent to correctly anticipate the reactions of his followers to his actions. In this paper we restrict ourselves to the subgraph information structure.

The specification of knowledge of the agents in the economy for the subgraph information structure corresponds to that of players in a multi-stage game with a backward induction solution in Bicchieri (1989). Therefore, in the case of an economy with a hierarchy tree, our equilibrium concept boils down to a subgame perfect equilibrium in an economy where the agent who is highest in the hierarchy moves first, the agents who are of one hierarchical level lower move next, etc., provided this game is well defined. There appears to be a trade off between the consistent economic interpretation of the specification of the model and the more abstract game theoretic requirements.

An equilibrium in a hierarchically structured trade economy is defined to be a tuple of actions of all agents such that these actions are feasible for every individual agent, and such that no agents anticipates to be possibly better of by choosing actions which are different from his equilbrium actions and which are feasible from his point of view. We prove a theorem on the existence of such equilibria for an economy with a hierarchy tree and the subgraph information structure with the rather restrictive assumption that the set of feasible choices of the topagent in the economy is non-empty.

We give three examples of hierarchically structured trade economies. In the first example we find that there does not exist an equilibrium because the set of feasible actions of the top agent in the economy is empty, although this economy has only three agents. From this example it also follows that the top agent in the economy may end up with a consumption bundle which yields him a lower utility level than his initial endowments. So it need not be advantageous to be the top-agent in the economy. The reason for this is that the top agent does not always trade voluntary because he has the obligation to meet the trades of his followers and he is the only agent in the economy that does not have the possibility to transfer theses trades to a leader.

The other two examples illustrate the impact the hierarchical structure of the economy may have on the equilibrium outcomes. For a set of three agents it shown what the equilbria in the economy are for different hierarchy trees.

From the results as derived in this paper we conclude that in a hierarchically structured trade economy with more than two hierarchical levels, a generic existence result for certain straightforward extensions of well accepted equilibrium concepts for the case of two echelons can no longer be proved. This especially illustrates that the structure of the market in well known models has important consequences for the existence of the equilibria for those situations. This shows that the trade – or social economic – structure of the economy is crucial in the description of economic trading processes.

This paper is organized as follows. In Section 2 we introduce the model, we derive the conjectures of the agents as a function of the information structure and we introduce the equilibrium concept. In Section 3 we prove the existence theorem. The examples are presented in Section 4 and in Section 5 some concluding remarks are made.

2 The Model

In this section we give a definition of hierarchically structured trade economies. For a discussion of the background of this approach we refer to Gilles (1989). A hierarchically structured trade economy is defined on the basis of an hierarchical structure which, in this paper, is assumed to be a hierarchy tree. A hierarchy tree is represented by a directed graph. We define the indegree of a point i in a directed graph $A, W)$, being the number of ingoing arrows of the point $i \in A$, by $\overleftarrow{\rho}(i) := \{(h,i) \in W \text{ with } h \in A\}$.

Definition 2.1 *A* **Hierarchy Tree** $\mathcal{T} := (A, W)$ *is a (weakly) connected, finite, simple directed graph which has a tree structure and is such that it has exactly one* $i \in A$ *such that* $\overleftarrow{\rho}(i) = 0$.

For each agent $i \in A$ we use $L_i := \{h \in A \mid (h,i) \in W\}$ to denote the set of *direct* leaders of agent i and we use $F_i := \{j \in A \mid (i,j) \in W\}$ to denote the set of *direct* followers of agent i.

A hierarchy tree is one of the possible hierarchical structures of an economy in which an arc from agent i to agent j means that agent i has the initiative over agent j. We say that i is the leader of agent j. So a source k in the graph (i.e. $\overleftarrow{\rho}(k) = 0$) represents an agent who has no leader. In the following we refer to agent k as the "top-agent" in the hierarchical structure.

The fact that the hierarchy tree has a tree structure and only one source implies that every agent i in the economy has at the most one (direct) leader. The hierarchy graph describes a partial order over the set of agents; it directs the set of agents.

Definition 2.2 *A* **Hierarchically Structured Trade Economy** *is a tuple* $\mathbf{E} = ((A, W), \{U_i, \omega_i\}_{i \in A})$, *where:*

1. (A, W) *is a hierarchy tree.*
2. $U_i : \mathbf{R}^l_+ \to \mathbf{R}$ *is the utility function of consumer* i *which is defined over an* l*-dimensional commodity space. The utility function is assumed to be strictly monotonic, continuous and strict quasi concave.*
3. $\omega_i \in \mathbf{R}^l_{++}$ *is the initial endowment of agent* i.
4. *We assume that each* $w = (i,j) \in W$ *is a monopolistic trade relation between the agents* i *and* j *where agent* i *is the price setter and agent* j *is the price taker with respect to this relation.*

One could argue, as in Hahn (1978) and, implicitly, in Vind (1983), that the conjectures of the agents about consequences of their actions, ought to be part of the description of the individual characteristics of the agents of the economy. Since we want to derive the conjectures of the agents as a function of the hierarchical structure, the (other) individual characteristics of the agents and of the institutional characteristics of the hierarchical relations, we refrain from making the conjectures of the agents an explicite part of the description of an economy.

We define a system of price vectors and trades of commodity bundles that we allow to occur in our trade economy. Let S^{l-1} be the $(l-1)$-dimensional unit simplex defined by

$$S^{l-1} := \{q \in \mathbf{R}^l_+ \mid \sum_{a=1}^{l} q_a = 1\}.$$

We show that we can restrict ourselves to a compact price space contained in int S^{l-1}, the relative interior of S^{l-1}.
Define $S_1 := \{a \in A \mid L_a = \emptyset\}$. In order to construct a suitable compact price space we consider, for each agent $i \in A \setminus S_1$ and for each $q \in \text{int } S^{l-1}$ the following optimization problem.

$$\max_{x_i \in \mathbf{R}^l_+} \; U_i(x_i)$$

subject to

$$q \cdot x_i \leq q \cdot \omega_i.$$

We denote the solution to this problem with $x_i^*(q, \omega_i)$. So $x_i^*(q, \omega_i)$ is the consumption planned by agent an $i \notin S_1$ given the initial endowment ω_i and the price vector q. Define for $\varepsilon > 0$:

$$Q := \{q \in S^{l-1} \mid q \geq \varepsilon \cdot 1_l\}.$$

We choose ε such that:

$$q \in S^{l-1} \setminus Q \Rightarrow \forall i \in A \setminus S_1 : \; \exists \; c \in \{1, \ldots, l\} : \; x_{ic}^*(q, \omega_i) > \omega_c,$$

where $\omega = \sum_{i \in A} \omega_i$. Such an ε exists by of the strict monotonicity of the utility functions of the agents. Denote $\tilde{\omega} := (\tilde{\omega}_c)_{c \in \{1,\ldots,l\}}$ where $\tilde{\omega}_c := \sum_{i \in A} \max_{q \in Q} x_{ic}^*(q, \omega)$.

We define the choice set of agent i to be $X_i = Y^{L_i} \times Q^{F_i}$, where $Y := \{d \in \mathbf{R}^l \mid -\tilde{\omega} \leq d \leq \tilde{\omega}\}$. Furthermore we denote

$$X := \prod_{i \in A} X_i.$$

We give the following definition of a trade-price-allocation-system in an given hierarchically structured trade economy. We assume ε to be as defined above.

Definition 2.3 *A* **Trade-Price-Allocation-System** *in the economy* $\mathbf{E}$ *is a pair* $(d, p, x) \in Y^W \times Q^W \times Y_+^A$ *where:*

1. $d_{ji} \in Y$ *denotes the trade on the trade relation* $(i, j) \in W$. *We define* $d_i := (d_{hi})_{h \in L_i}$.
2. $p_{ij} \in Q$ *is the price system denoting the prices charged on the trade-relation* $(i, j) \in W$. *We define* $p_i := (p_{ij})_{j \in F_i}$.
3. $x_i \in Y_+ := Y \cap \mathbb{R}_+^l$ *is the consumption bundle for agent* i.

Agents assume that their actions do not influence the prices their leader sets for them. On the other hand we assume that each agent correctly anticipates the consequences for the demand of his followers as a function of the prices he sets. We assume him to take into account the consequences possible changes in the prices these followers set for the actions of their followers etc. This behaviour may occur if each agent has perfect information about the subgraph starting from him and knows nothing but the price his leader set for him, about the rest of the economy. Thus we assume the subgraph of each agent is perfectly transparent for this agent, or at least has the information that summarizes what trades come from his subgraph as a function of the prices the agent sets for his followers. We call this specification of knowledge the **subgraph information structure**.

The conjectures of the agents are described by their anticipated trade correspondences. These anticipated trade correspondences are defined recursively, starting with the agents $j \in A_0 := \{j \in A \mid F_j = \emptyset\}$, who do not have any followers. Then, given the anticipated trade correspondences for some set of agents A_t with $t \in \mathbb{N}$, we derive the anticipated trade correspondences for the set of agents $C_{t+1} := \{i \in A \setminus B_t \mid F_i \subset A_t\}$. Then we define $A_{t+1} := A_t \cup C_{t+1}$ etc. We stop this procedure when we have a t^* such that $A_{t^*} = A$. Since $\mathcal{H}$ is a hierarchy tree such a t^* exists. The aniticipated trade correspondences can thus derived using the following definition. Note that, in this special case, any anticipated trade correspondencs t_{ij} is exactly the reaction correspondence of agent j with repect to the trades with agent i.

Definition 2.4 *The* **Anticipated Trade Corrrespondence** $t_{ij} : Q \rightrightarrows Y$ *of agent i with respect to agent $j \in F_i$ under subtree information is defined to be such that*

$$\begin{aligned} t_{ij}(q_{ij}) \quad = \quad & \text{argmax}_{e_{ji} \in Y} \{ \max_{q_j \in Q^{F_j}} \{ U_j(e_{ji} + \omega_j - \sum_{m \in F_j} e_{mj}) \mid \\ & q_{ij} \cdot e_{ji} \leq 0 \quad \text{and} \quad \forall m \in F_j : \; e_{mj} \in t_{jm}(q_{jm}) \} \}. \end{aligned}$$

Now we have defined the anticipated trade correspondences we define the budget correspondence of some agent i with $L_i \neq \emptyset$ to be such that given the prices set by his leader the corresponding budget set contains the actions agent i anticipates to be feasible. We define the budget set of the top-agent to be the set of actions he anticipated to be feasible.

Definition 2.5 *The* **Budget Correspondence** $B_i : Q \rightrightarrows X_i \times Y_+$ *of agent i with $L_i = \{h\}$ under the subgraph information structure is defined as:*

$$\begin{aligned} B_i(p_{hi}) := \{ (e_{ih}, q_i, y_i) \in X_i \times Y_+ \quad \mid \quad & e_{ih} \cdot p_{hi} \leq 0 \\ & \text{and} \quad y_i \; \leq \; \omega_i \; + \; e_{ih} \; - \; \sum_{j \in F_i} e_{ji} \\ & \text{with} \quad e_{ji} \in t_{ij}(q_{ij}) \; \}. \end{aligned}$$

The budget set for the top-agent, i.e. the agent k with $L_k = \emptyset$, is defined as:

$$\begin{aligned} B_k := \{ (q_k, y_k) \in X_k \times Y_+ \quad \mid \quad & y_k \; \leq \; \omega_k \; - \; \sum_{j \in F_k} e_{jk} \\ & \text{with} \quad e_{jk} \in t_{kj}(q_{kj}) \; \}. \end{aligned}$$

The optimization problem of agent i who has some agent h as his direct leader is to maximize his utility over his budget set. Here his budget set depends on the prices p_{hi} his leader sets for him. Therefore agent i solves the following optimization problem:

$$\max_{(e_{ih}, q_i, y_i) \in B_i(p_{hi})} U_i(y_i).$$

The optimization problem of the top-agent k in the economy is

$$\max_{(q_k, y_k) \in B_k} U_k(y_k).$$

Now we have described the individual optimization problem for every agent i we define an equilibrium in the hierarchically structured trade economy in the case of subtree information.

Definition 2.6 *A tuple* $(d^*, p^*, x^*) \in X \times Y_+$ *is an* **equilibrium** *in the trade economy* $\mathbf{E}$ *with subtree information if* $\forall\, i \in A$ *such that* $\exists\, h \in A$ *with* $L_i = \{h\}$*:*

1. $(d_i^*, p_i^*, x_i^*) \in B_i(p_{hi}^*)$.
2. $\nexists (e_{ih}, q_i, y_i) \in B_i(p_{hi}^*)$ *such that* $U_i(y_i) > U_i(x_i^*)$.

and for the top-agent k *we have that*

1. $(p_k^*, x_k^*) \in B_k$.
2. $\nexists (q_k, y_k) \in B_k$ *such that* $U_k(y_k) > U_k(x_k^*)$.

So an equilibrium is a tuple of actions of the agents in the economy such that:

1. *[Feasibility]* The tuple is anitcipated to be feasible from the point of view of each agent.
2. *[Stability]* No agent can choose his actions, in a manner feasible given the equilibrium tuple, in such a way that he anticipates this change in actions to (possibly) make him better off.

3 Existence of Equilibrium

In this section we prove a theorem on the existence of equilibrium in a hierarchically structured trade economy with a hierarchy tree and the subgraph information structure. First, in Lemma 3.1, we prove all anticipated trade correspondences have compact graphs. In the proof of Theorem 3.2, the equilibrium existence theorem, we use this lemma and we apply Weierstrass' Theorem.

As follows from the definition of the anticipated trade correspondences and our equilibrium concept, we find that the equilibria in the hierarchically structured trade economy correspond to the subgame perfect equilibria in the following game and vice versa, if the game is well defined. In Stage 1 of the game the top agent sets the prices for his followers. In Stage 2 these followers determine the amounts they want to trade with the top agents and simultaneously they set the prices for trade with their followers, and so on for the next stages until each agent in the economy has made his moves. Indeed, if the corresponding game would always be well defined, we could have defined the subgraph information structure as arising from a multi-stage game.

This property might lead to the idea to prove Theorem 3.2 by using the existence theorem of Harris (1985) or the theorem of Hellwig and Leininger (1987). Unfortunately the budget correspondences as defined in Section 2 do not satisfy the assumptions of this theorem. Nevertheless, by using some special properties of our model we can proof an existence theorem. Unfortunately the conditions of the existence theorem are rather restrictive and boil down to to the corresponding multi-stage game being well defined.

The conditions for existence of equilibrium are satisfied, however, if for each agent who has the top-agent as his direct leader, it holds that the function which describes the profits an agent can make due to his position as a middleman is single peaked for every price his leader may set. This type of assumption is, amongst others, made in Marschak and Selten (1974) and Krelle (1976).

In the Lemma 3.1 we make use of the property that we can split the optimization problem of an agent $i \in A$ with $L_i \neq \emptyset$ in maximizing his profits from trade and optimizing his consumption given his income. The profits from trade of agent i are the profits he can make because of his position as a middleman. The profits from trade and his initial endowments determine the income of agent i, which determines his set of affortable consumption bundles over which he maximizes his utility function.

Lemma 3.1 *$\forall a \in A,\ j \in F_i$ it holds that t_{ij} has a compact graph.*

Proof
We prove this theorem by induction.
Starting Condition: Suppose $F_j = \emptyset,\ L_j = \{i\}$. Then t_{ij} has a compact graph. This follows directly from applying the Maximum Theorem.
Induction Hypothesis: Suppose $F_j \neq \emptyset,\ L_j = \{i\}$. Suppose that for each $m \in F_j$ it holds that t_{jm} has a compact graph. Then it holds that t_{ij} has a compact graph.
Proof of the Induction Hypothesis
Since X is compact we must show that if $p_{ij}^n \in Q$ and $e_{ji}^n \in Y$ for each $n \in \mathbb{N}$ such that

$$\{p_{ij}^n\}_{n=1}^{\infty} \to p_{ij}^0, \quad \{e_{ji}^n\}_{n=1}^{\infty} \to e_{ji}^0$$

and

$$\forall n \in \mathbb{N} \setminus \{0\} : \ e_{ji} \in t_{ij}(p_{ij}^n)$$

then

$$e^0_{ji} \in t_{ij}(p^0_{ij}).$$

Define $\pi_j : Q \rightrightarrows \mathbf{R}$ to be such that $\pi_j(p_{ij})$ is that set of values of π of the solutions, given p_{ij}, of

$$\max_{(q_{jm},e_{mj})_{m\in F_j}\in(Q\times Y)^{F_j}} \pi := \sum_{m\in F_j}(q_{jm} - p_{ij})\cdot e_{mj}$$

such that

$$e_{mj} \in t_{jm}(q_{jm}) \qquad \text{for each } m \in F_j.$$

$\pi_j(p_{ij})$ is the maximal profit from trade that agent j can induce given p_{ij}, evaluated at prices p_{ij}. Since $\forall\, m \in F_j$ the correspondence t_{jm} has a compact graph by assumption, it follows by the Maximum Theorem that π_j can be represented by a continuous function. Furthermore it holds for all $p_{ij} \in Q$ that $\pi_j(p_{ij}) \geq 0$. Define the correspondence $\mu_j : Q \rightrightarrows (Q\times Y)^{F_j}$ such that $(q_{jm}, e_{mj})_{m\in F_j} \in \mu_j(p_{ij})$ if and only if the tuple $(q_{jm}, e_{mj})_{m\in F_j}$ maximizes the profits for trade of agent j given p_{ij}. From the Maximum Theorem it follows that μ_j has a compact graph.

The optimal consumption bundles for agent j at prices $p_{ij} \in Q$, as they follow from the optimization problem of agent j as described in Section 2, result as solutions of

$$\max_{y_j\in Y_+} U_j(y_j)$$

such that

$$p_{ij}\cdot y_j \leq \pi_j(p_{ij}) + p_{ij}\cdot\omega_j,$$

and vice versa.

By the continuity of the innerproduct for $p_{ij} \in Q$, since U_j is a continuous function and since π_j is a continuous non-negative function, we can apply the Maximum Theorem. Since U_j is strict quasi-concave and since the set of feasible y_j is convex for given p_{ij}, we can represent the solutions of the problem by a continuous function $x_j : Q \rightarrow Y_+$ of p_{ij} such that $x_j(p_{ij})$ is the optimal attainable consumption bundle for agent j.

By definition the trades $e^n_{ji} \in t_{ij}(p_{ij})$ follow from solving the optimization problem of agent j as described in Section 2. Using the strict monotonicity of U_j we find that this is equivalent to stating that for each $n \in \mathbb{N}$ there exist $(q^n_{jm}, e^n_{mj})_{m \in F_j} \in \mu_j(p^n_{ij})$ such that

$$e^n_{ji} = \sum_{m \in F_j} e_{mj} + x_j(p^n_{ij}) - \omega_j \qquad \text{for all } n \in \mathbb{N} \setminus \{0\}.$$

By the continuity of x_j, by the continuity of the summation and since μ_j has a compact graph we have that $\exists\,(q^0_{jm}, e^0_{mj})_{m \in F_j} \in \mu_j$ such that

$$e^0_{ji} = \sum_{m \in F_j} e_{mj} + x_j(p^0_{ij}) - \omega_j.$$

But this is equivalent with $e^0_{ji} \in t_{ij}(p^0_{ij})$.

The fact that $\mathcal{T}$ is a hierarchy tree completes the proof.
$\mathcal{Q.E.D.}$

Theorem 3.2 [Existence Theorem]
An equilibrium in $\mathbf{E}$ *exists if and only if for the top-agent* $k \in S_1$ *we have that* $B_k \neq \emptyset$.

Proof

Only if

Follows directly from Definition 2.6 .

If

First we show that the budget correspondence B_i has compact and non-empty values for every $i \in A$. Since by Lemma 3.1 we have that for all $i \in A$, $j \in F_j$ the anticipated trade correspondence t_{ij} has a compact graph, the budget correspondences B_i have compact values. Let $i \in A$, $L_i \neq \emptyset$. Let $h \in L_i$. Now for every $p_{hi} \in Q$ it holds that $(e_{ih}, q_i, y_i) \in B_i(p_{hi})$ for $e_{ih} := \sum_{j \in F_i} e_{ji}$ with $e_{ji} \in t_{ij}(p_{hi})$, $q_i := (p_{hi})_{j \in F_i}$ and $y_i := \omega_i$. Therefore B_i does not have empty values.
Furthermore if $L_i = \emptyset$, then by assumption we have that $B_i \neq \emptyset$. Since $\mathcal{H}$ is a hierarchy graph we have by Backward Induction (repeatedly applying Weierstrass' Theorem) that an equilibrium in $\mathbf{E}$ exists.

$\mathcal{Q.E.D.}$

The following corollary states that if for each of the direct followers of the top-agent k it holds that the profits from trade function is single peaked for each price vector agent k may choose to set. This is a common assumption in the realm of successive monopolies. This type of assumption can, amongst others, be found in Marschak and Selten (1974) and Krelle (1976).

Corollary 3.3 *If for each agent i with $F_i \neq \emptyset$ and $L_i = \{k\}$, with $L_k = \emptyset$, it holds that for all $p_{hi} \in Q$ we have that $\mu_i(p_{hi})$ is singleton valued, then an equilibrium in* **E** *exists.*

Proof
By Lemma 3.1 we know that t_{ki} has a compact graph. By the singleton valuedness of μ_i and the strict quasi concavity of U_i we know that t_{ki} must be singleton valued and therefore corresponds to a continuous function. Since X_k is a convex set it follows by the Mean Value Theorem that there exists a system of prices $(q_{ki})_{i\in F_k}$ such that $\sum_{i\in F_k} t_{ki}(q_{ki}) = 0$. Therefore we have that B_k is non-empty and thus by Theorem 3.2 we know that an equilibrium in **E** exists.
$\mathcal{Q.E.D.}$

4 Some Examples

In this section we give three examples of trade economies. The first example illustrates the possible non-existence of equilibrium in a trade economy with subgraph information. In this example we do not make explicite use of the specific utility functions and intitial endowments of the agents in the economy. The chosen specifications are made in order to simplify the calculations.

In the other two examples we investigate the influence of different hierarchy trees on the equilibrium outcomes in the economy. In order to do so we consider an economy with two commodities and three agents for which we calculated the equilibria for different hierarchy trees. We investigate whether some hierarchy trees lead to more efficient equilibrium outcomes in the economies than others.

Example 4.1
Consider economy with two goods, commodity 1 and commodity 2, and three

agents. Assume the hierarchy tree is $\mathcal{T} := (\{a,b,c\},\{(a,b),(b,c)\})$, i.e. agent a is the top agent, agent b is the middle-man and agent c is of the lowest hierarchical level. We use p_1 as the price of commodity 1. Given p_1 the price p_2 of commodity 2 follows by $p_2 = 1 - p_1$. Agent c has a net-demand function which has the following form on the interval $[\frac{1}{2}, \frac{3}{4}]$.

$$d_{c1}(p_{b1}) = \begin{cases} 32p_{b1} - 16 & p_{b1} \in [\frac{1}{2}, \frac{9}{16}\rangle. \\ 16p_{b1} - 7 & p_{b1} \in [\frac{9}{16}, \frac{5}{8}\rangle. \\ 3 & p_{b1} \in [\frac{5}{8}, \frac{21}{32}\rangle. \\ 64p_{b1} - 39 & p_{b1} \in [\frac{21}{32}, \frac{11}{16}\rangle. \\ 5 & p_{b1} \in [\frac{11}{16}, \frac{3}{4}]. \end{cases}$$

This net-demand function is continuous and monotonous on the interval $[\frac{1}{2}, \frac{3}{4}]$. From the strict monotonicity of the utility functions it follows that the total value of the trade over each trade relation will be zero, so the net-demand function of agent c for commodity 2 is:

$$d_{c2}(p_{b1}) = \frac{-p_{b1}}{1 - p_{b1}} \cdot d_{c1}(p_{b1}).$$

The functions d_{c1} and d_{c2} are depicted in Figure 1.

The optimization problem of agent b can be separated in maximizing the (anticipated) income from the trade with agent c, and maximizing his utility given his total wealth at the prices set for him by agent a.

The function denoting the income from trade for agent b at price p_{b1}, given the net-demand functions of agent c and given the price p_{a1} set by agent a is

$$\begin{aligned} \pi(p_{a1}, p_{b1}) &= (p_{b1} - p_{a1})d_{c1}(p_{b1}) + ((1 - p_{b1}) - (1 - p_{a1}))d_2(p_{b1}) \\ &= (p_{b1} - p_{a1})d_{c1}(p_{b1}) + (p_{a1} - p_{b1})\tfrac{-p_{b1}}{1-p_{b1}} \cdot d_{c1}(p_{b1}) \\ &= (p_{b1} - p_{a1})(\tfrac{p_{b1}}{1-p_{b1}} + 1)d_{c1}(p_{b1}) \\ &= (p_{b1} - p_{a1})\tfrac{1}{1-p_{b1}} d_{c1}(p_{b1}). \end{aligned}$$

The (anticipated) income from trade for agent b as a function of p_{b1} for $p_{a1} = \frac{3}{4}$ is drawn in Figure 2. The net-demand function of agent c corresponds to the anticipated trade correspondence t_{bc}.

The income from trade function of agent b has two global maxima, each corresponding to a different anticipated trade between agent c and agent b. The value

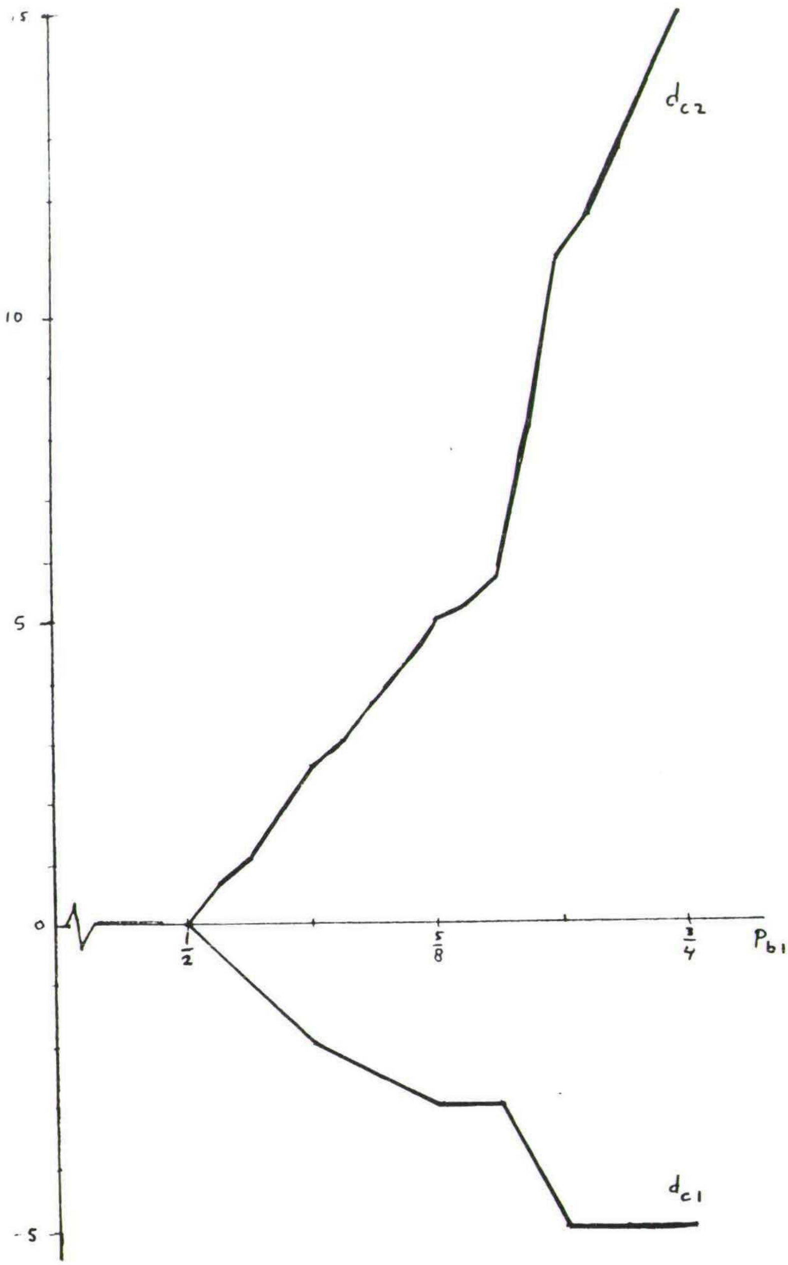

FIGURE 1: The net-demand functions of agent c.

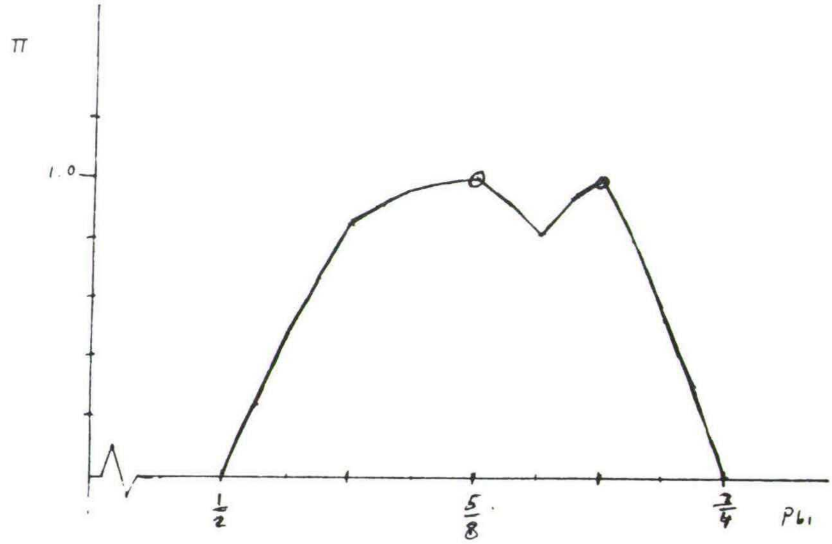

FIGURE 2: The income from trade function $\pi(\frac{3}{4}, p_{b1})$.

of the income from trade function in each of the maxima is 1, the corresponding prices p_{b1} are $p_{b1} = \frac{5}{8}$ and $p_{b1} = \frac{11}{16}$.

Figure 3 and Figure 4 show the income from trade function of agent b for the prices $p_{a1} = \frac{23}{32}$ and $p_{a1} = \frac{25}{32}$. For each of these prices p_{a1} there exists a unique global maximum of the income from trade function of agent b, although there are two local maxima. For $p_{a1} < \frac{3}{4}$ the anticipated trade of agent c in commodity 1

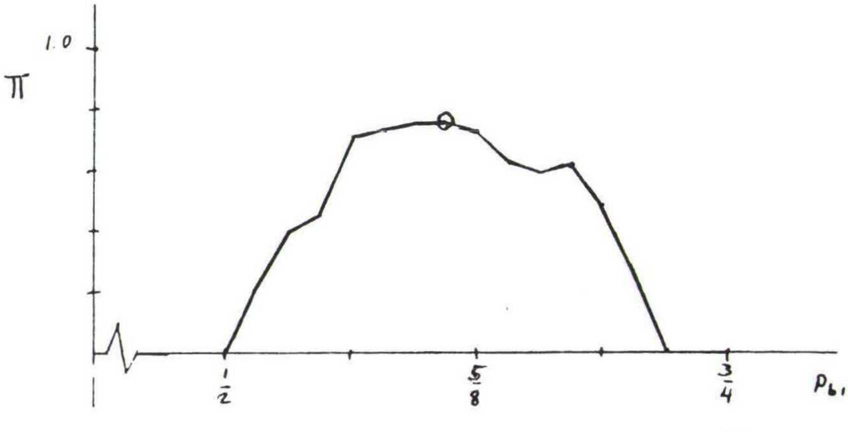

FIGURE 3: The income from trade function $\pi(\frac{23}{32}, p_{b1})$.

at the price that maximizes the income from trade for agent b, exceeds the largest

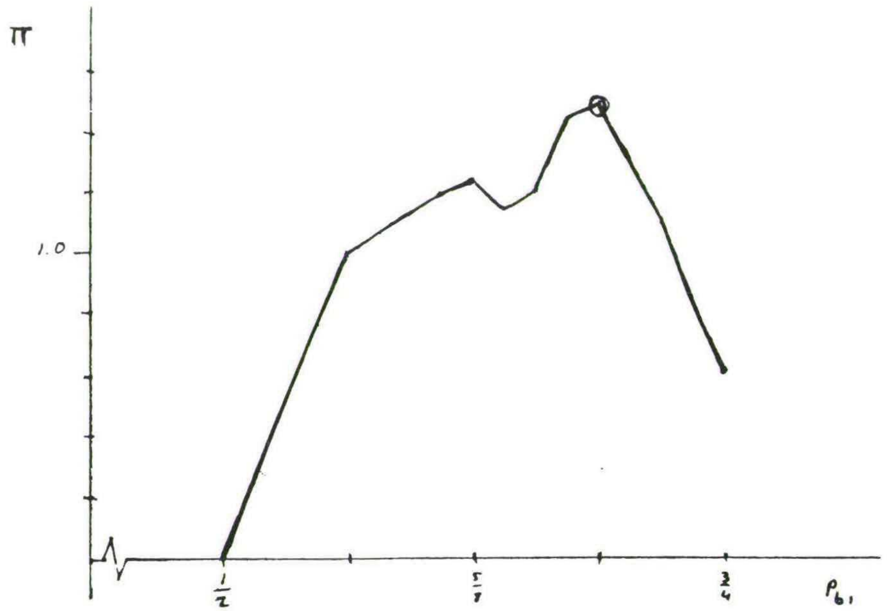

FIGURE 4: The income from trade function $\pi(\frac{25}{32}, p_{b1})$.

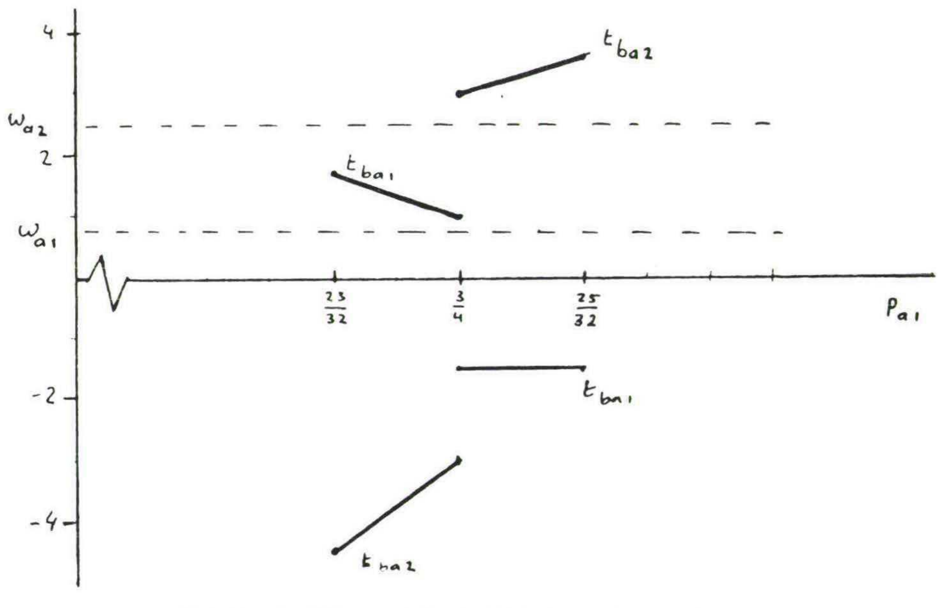

FIGURE 5: The anticipated trade correspondence t_{ab}.

of the trades for the the prices that maximize the income from trade for agent b at $p_{a1} = \frac{3}{4}$. For $p_{a1} > \frac{3}{4}$ the anticipated trades of agent c in commodity 1 at the prices that maximize the income from trade for agent b are smaller than the smallest of the trades for the the prices that maximize the income from trade for agents b at $p_{a1} = \frac{3}{4}$.

Now suppose that agent b has a net-demand for commodity 1 for his own consumption of 4 units, for each of the prices $p_{a1} \in [\frac{5}{8}, \frac{7}{8}]$, independent of the income from the trade with agent c. Using the budget constraint and the strict monotonicity of the utility functions we arrive at the following net-demand function of agent b as function of the prices following from p_{a1}, disregarding the amounts of the commodities necessary for the trade with agent c:

$$\begin{aligned} d_b(p_{a1}) &= \tfrac{-p_{a1}}{1-p_{a1}} \cdot d_{b1}(p_{a1}) \\ &= \tfrac{-3p_{a1}}{1-p_{a1}}. \end{aligned}$$

This leads to the trade correspondence of agent b as depicted in Figure 5. This is exactly the anticipated trade correspondence t_{ab}. Lemma 3.1 and the Maximum Theorem ensure this correspondence is upper hemi-continuous, but it need not be singleton valued. The non-convexity in the value of the anticipated trade correspondence at $p_{a1} = \frac{3}{4}$ is caused by the existence of two global maxima of the income from trade function of agent b at price $p_{a1} = \frac{3}{4}$.

Assume agent a has relatively small endowments of the commodities 1 and 2 as in Figure 5. We safely assume that the total net-demand of agent b for commodity 1 does not decrease as p_{a1} decreases and that it does not increase as p_{a1} increases. Now the non-convexity in the value of the anticipated trade correspondence makes it impossible for agent a to set a price p_{a1} such that he can supply the amount he (correctly) anticipates agent b to order in their trade relation at p_{a1}. But then no equilibrium in the economy as in this example exists.

Example 4.2

Consider an economy with two goods, commodity 1 and commodity 2, which consists of three agents, a, b and c, i.e. $A = \{a, b, c\}$. We assume commodity 1 to be the numeraire and we assume

$$\begin{aligned} &U_a(x_a) = (\sqrt{x_{a1}} + \sqrt{x_{a2}})^2, && \omega_a = (0,0) \\ &U_b(x_b) = (\sqrt{x_{b1}} + \sqrt{x_{b2}})^2, && \omega_b = (1,0) \\ &U_c(x_c) = (\sqrt{x_{c1}} + \sqrt{x_{c2}})^2, && \omega_c = (0,1) \end{aligned}$$

With respect to this set of agents we consider the following three different hierarchy trees:

$$\mathcal{T}_1 := (A, \{(a,b),(b,c)\})$$
$$\mathcal{T}_2 := (A, \{(a,b),(a,c)\})$$
$$\mathcal{T}_3 := (A, \{(b,a),(a,c)\})$$

In the economy which has $\mathcal{T}_1$ as its hierarcy tree we use p to denote p_{ab} and we use q to denote p_{bc}, for $\mathcal{T}_2$ we use p to denote p_{ab} and q to denote p_{ac} and finally for $\mathcal{T}_3$ we use p to denote p_{ba} and q to denote p_{ac}.

This example is such that an allocation is Pareto efficient if and only if $\sum_{i\in A} U_i(x_i^*) = 4$. So we may represent the efficiency of the organization as it follows from a hierarchy tree $\mathcal{T}$ which has x^* as its equilibrium allocation as $\text{Eff}(\mathcal{T}) := \frac{1}{4}\sum_{i\in A} U_i(x_i^*)$. Thus we find the equilibria as in Table I.

For $\mathcal{T}_1$ we find that the equilibrium corresponds to the duopoly outcome where

TABLE I: Equilibrium Values

Variable	$\mathcal{T}_1$	$\mathcal{T}_2$	$\mathcal{T}_3$
p^*	1.4740	0.4142	0.9113
q^*	0.5729	2.4142	0.3825
x_{a1}^*	0	0.1716	0.0698
x_{a2}^*	0	0.1716	0.0840
x_{b1}^*	0.7913	0.7071	0.8244
x_{b2}^*	0.3642	0.1213	0.1927
x_{c1}^*	0.2087	0.1213	0.1058
x_{c2}^*	0.6358	0.7071	0.7233
$U_a(x_a^*)$	0	0.6864	0.3069
$U_b(x_b^*)$	2.2290	1.4142	1.8141
$U_a(x_c^*)$	1.5730	1.4142	1.3823
Efficiency	95.05%	87.87%	87.58%

agent b dominates agent c. If we delete agent a from the economy the equilibrium outcome does not change. As Example 4.1 shows us, this does not hold in general, we may have individual characteristics of the agents b and c such that no equilibrium exists if agent a has $(0,0)$ as his vector of initial endowments. We find that in an economy with three agents and a structure as above, a top-agent with zero initial endowments either does not matter, or is the cause of non existence of equilibrium.

In the economy which has $\mathcal{T}_2$ as its hierarchy tree we find that agent a has the position of a monopolist who can perform price discrimination between the agents b and c. The consumption of agent a is non zero because his power as an intermediary allows him to make profits from trade. Therefore we can interpret the level of utility agent a achieves as a measure of the value of his position as intermediary.

For $\mathcal{T}_3$ we find that the consumption of agent a lower than it was for $\mathcal{T}_2$. So his position as an intermediary in this structure is of less value than it was in the second example.

We find that $\mathcal{T}_1$ is the most efficient hierarchy tree for this example, although for none of the structures the outcomes are prefered by all agents to the outcomes of another structure. Therefore we may say that in this example non of the above three structures dominates another of these structures. The efficiency attained for $\mathcal{T}_1$ in this example is also attained for the hierarchy trees which have as their set of arrows $\{(a,c),(c,b)\}$ $\{(b,c),(c,a)\}$ $\{(c,b),(b,a)\}$ $\{(b,c),(b,a)\}$ or $\{(c,b),(c,a)\}$.

Example 4.3

Consider an economy $\mathbf{E}$ with two goods, commodity 1 and commodity 2, such that $\mathbf{E}:= (\{a,b,c\},\{(a,b),(a,c)\})$. We assume commodity 1 to be the numeraire and we assume:

$$\begin{aligned} U_a(x_a) &= x_{a1} + x_{a2} & \qquad \omega_a &= (0,0) \\ U_b(x_b) &= x_{b1} + x_{b2} & \qquad \omega_b &= (1,0) \\ U_c(x_c) &= x_{c1} + x_{c2} & \qquad \omega_c &= (0,1) \end{aligned}$$

As in Example 4.2 we consider the following three different hierarchy trees with respect to this set of agents.

$$\begin{aligned} \mathcal{T}_1 &:= (A, \{(a,b),(b,c)\}) \\ \mathcal{T}_2 &:= (A, \{(a,b),(a,c)\}) \\ \mathcal{T}_3 &:= (A, \{(b,a),(a,c)\}) \end{aligned}$$

In the economy which has $\mathcal{T}_1$ as its hierarcy tree we use p to denote p_{ab} and we use q to denote p_{bc}, for $\mathcal{T}_2$ we use p to denote p_{ab} and q to denote p_{ac} and finally for $\mathcal{T}_3$ we use p to denote p_{ba} and q to denote p_{ac}.

Although we refrain form defining efficiency in an exact manner, we maintain that we take Pareto efficient outcomes to have an efficiency rate of 100%. In this

TABLE II: Equilbrium Values

Variable	$\mathcal{T}_1$	$\mathcal{T}_2$	$\mathcal{T}_3$
p^*	0.3333	0.3333	0.3333
q^*	0.6667	0.6667	0.6667
x^*_{a1}	0	0.5000	0
x^*_{a2}	0	0.5000	0
x^*_{b1}	0.5000	0	0.5000
x^*_{b2}	1	0.5000	1
x^*_{c1}	0.5000	0.5000	0.5000
x^*_{c2}	0	0	0
$U_a(x^*_{a1})$	0	1.25	0
$U_b(x^*_{b1})$	2.50	1	2.50
$U_c(x^*_{c1})$	1	1	1
Efficiency	100%	100%	100%

example, contrary to the previous example, for each of the hierarchy trees the equilibrium outcomes are Pareto efficient, and therfore each of these equilibia has an efficiency of 100%. In Table II we find that in the economy with $\mathcal{T}_3$ the intermediary, agent a, does not have any advantage from his position as a middleman. His presence does not even influence the outcome as compared to the duopoly outcome for the agents b and c with agent b dominating agent c.

In the economy with $\mathcal{T}_2$, however, we find that agent a, in his position as a price differentiating monopolist, can claim the entire surplus from trade. In this economy his position as an intermediary is very valuable indeed.

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